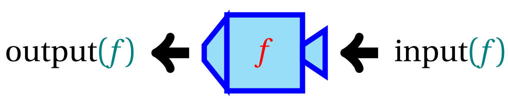
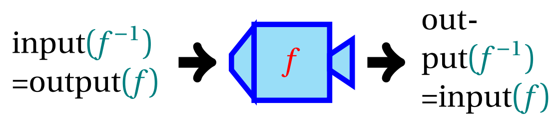
1.7: Inverse Functions





One-to-one functions

- A function is one-to-one if every output is produced from only one input.
- Horizontal line test: If every horizontal line intersects the graph of a function at most once, then the function is one-to-one.
- If a function is one-to-one then the inverse relation is a function as well.
- The inverse function of a one-to-one function $y = f(x)$, denoted by f^{-1}, is $x = f^{-1}(y)$.

Properties of Inverse Functions

- If f^{-1} is the inverse of f, then $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$. That is, their composition is the identity function.
- If f^{-1} is the inverse of f, then f is also inverse of f^{-1}. That is, $\left(f^{-1}\right)^{-1}(x) = f(x)$.
- (Domain of f)=(Range of f^{-1}) and (Range of f)=(Domain of f^{-1}).

Restricted Domains

- If a function is not one-to-one, we may be able to restrict its domain to obtain a one-to-one function. This way, we can find an inverse function for that piece. For functions without any breaks (continuous functions), this means that we are finding domain of the largest increasing piece or the largest decreasing piece of the graph.

How to Find the Rule of Inverse Function

- Choose an output variable and set equal to the rule of the function. (For example, $y = f(x)$.)
- Solve for the input variable (for example, x) in terms of the output variable (for example, y)
- Interchange the input variable and output variable. The output variable is now $f^{-1}(x)$.

Comparing graphs

- Graph of the inverse of a function is the reflection of the function over the $y = x$ axis. This means if (x, y) belongs to graph of f, then (y, x) belongs to graph of f^{-1}.

1. Use horizontal line test to identify the graphs of one-to-one functions.

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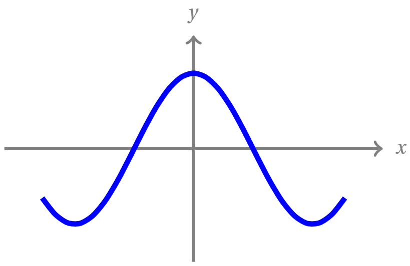

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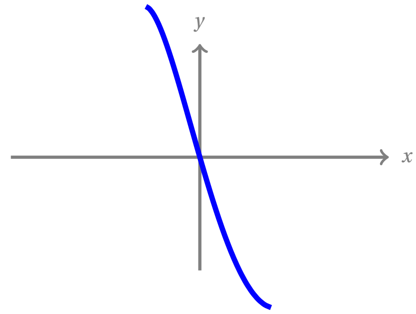

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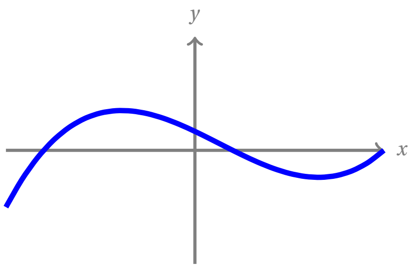

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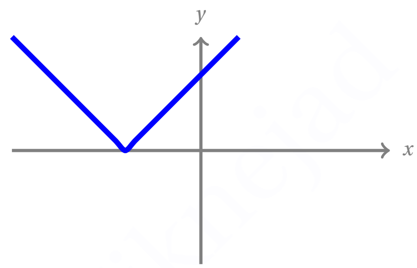

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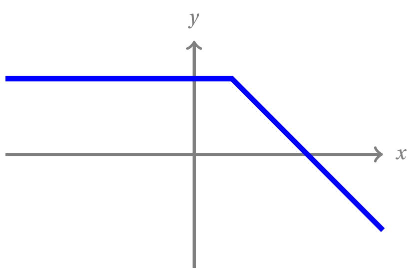

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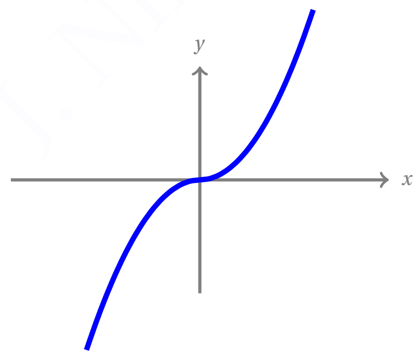


2. The formula to convert degrees Celsius to degrees in Fahrenheit is $F = \frac{9}{5}C + 32$. This is considered to be the function $F(C) = \frac{9}{5}C + 32$, where input is C, temperature in degrees in Celsius and output is F, temperature in degrees in Fahrenheit.

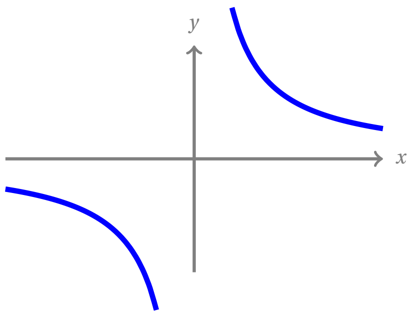
(A) Solve for C in $F = \frac{9}{5}C + 32$.

(B) Represent your solution in Part (A) as $C(F)$. What is the input and output of $C(F)$?

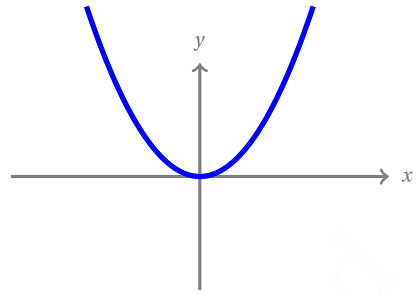
(C) Note that $C(F)$ is the inverse of $F(C)$. What is the interpretation of this inverse function?

3. For each of the graphs below, say that the function is one-to-one or restrict the domain of the function so the resulting function is a one-to-one function. Then find the inverse of the function. Find the domain and the range of the inverse.

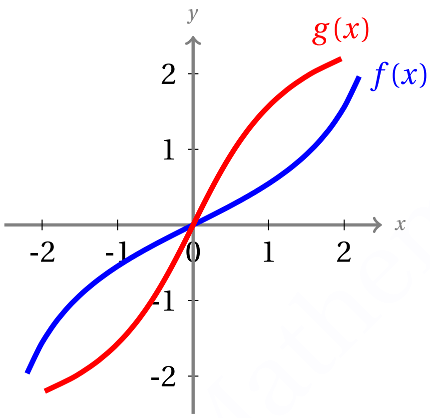
(a) $g(x) = \dfrac{1}{x}$.



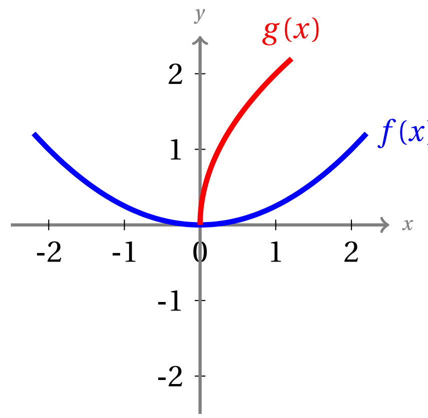
(b) $f(x) = x^2$



4. Which of the following pairs of graphs represent a function, f, and its inverse function, g? For pair(s) that are not inverse of each other, can you restrict function f so they become inverse of each other?



(a)



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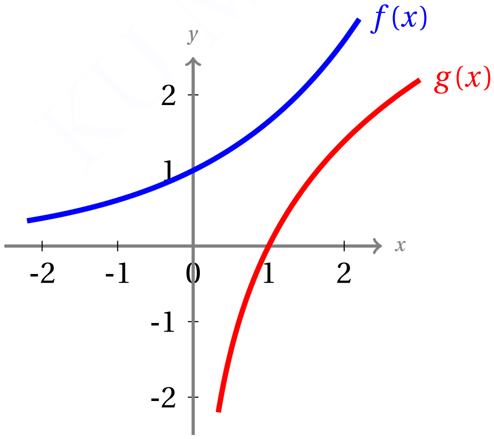
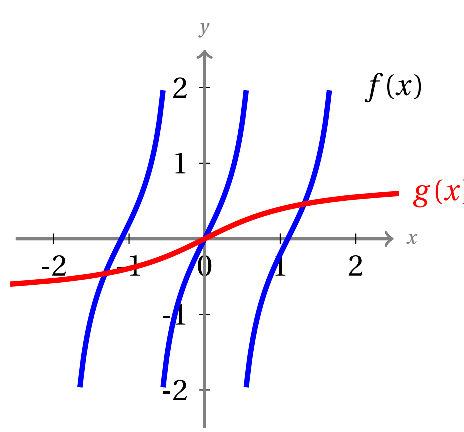


(b)



(d)

5. Some values of invertible functions f and g are given in the table to the right. Find the following values.

x	$f(x)$	$g(x)$
1	3	3
2	1	4
3	4	2
4	2	1

(a) $f^{-1}(4)$

(b) $f^{-1}(3)$

(c) $g^{-1}(1)$

(d) $g^{-1}(2)$

(e) $(f \circ g^{-1})(2)$

(f) $(g^{-1} \circ f)(4)$

6. Find the inverse of $f(x) = 4x + 3$.

7. Find the inverse function of $f(x) = \dfrac{1}{3x+2}$.

8. Find the inverse function of $f(s) = \dfrac{-7}{3s+2}$.

9. Find the inverse function of $m(t) = \dfrac{2t+5}{3t}$.

10. Find the inverse function of $v(t) = \dfrac{7t+2}{3t-5}$.

Related Videos to Inverse Computation

1. Watch Gateway Video 86: https://mediahub.ku.edu/media/MATH+104+-+086/1_b3ptd7de
2. Watch Gateway Video 87: https://mediahub.ku.edu/media/MATH+104+-+087/1_xhnunjds
3. Watch Gateway Video 88: https://mediahub.ku.edu/media/MATH+104+-+088/1_zca1yifi
4. Watch Gateway Video 89: https://mediahub.ku.edu/media/MATH+104+-+089/1_86nenuna
5. Watch Gateway Video 90: https://mediahub.ku.edu/media/MATH+104+-+090/1_bw14it8x
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8. Watch Gateway Video 93: https://mediahub.ku.edu/media/MATH+104+-+093/1_9ylzufn5
9. Watch Gateway Video 94: https://mediahub.ku.edu/media/MATH+104+-+094/1_281vuxy4
10. Watch Gateway Video 95: https://mediahub.ku.edu/media/MATH+104+-+095/1_gqvy2ffh

In some literature, equations with many letters are called **literal equations**. Solving a literal equation fro one of the letters is very much the process you used in finding inverse functions.

Solving Equations with Multiple Parameters: **PreCalculus Version)**
If the desired variable only appears to power of one, then follow the following process.

Isolate the Variable: First manipulate both sides so that each side clearly consists of different terms. For example, if one or both sides are quotient expressions, multiply both sides by each factor in denominator, Multiply all factors through and eliminate square roots. Add or subtract terms on both sides of the equation, make all terms on one sides contain the desirable variable and all terms on the other side do not contain that variable.

Factor the Variable: If the desirable variable still appears to power one only, you can factor the variable on one side.

Divide: Divide both sides by what multiplied the desirable variable.

(L1) Solve $P = S - Srt$ for r. (Watch Video 11.)
(This is an equation of linear decay.)

(L2) Solve $2rx + 5 = 6(r - x)$ for x. (Watch Video 12.)

(L3) Solve $v = \dfrac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$ for v_1. (Watch Video 15.)
(This is the formula for adding two large velocities; according to the theory of relativity.)

(L4) Solve $x + y = \sqrt{x^2 + y^2 + 3}$ for y. (Watch Video 17.)

(L5) Solve $Q_\omega = m_\omega c_\omega (T_f - T_\omega)$ for T_w. (Watch Video 18.)
(This is a thermal equilibrium equation.)

(L6) Solve $y - y_1 = m(x - x_1)$ for x. (Watch Video 19.)
(This is an equation of a line.)

(L7) Solve $\dfrac{x}{a} + \dfrac{y}{b} = 1$ for x. (Watch Video 21.)
(This is another equation for a line in x, y.)

(L8) Solve $\dfrac{1}{x} + \dfrac{1}{y} = 1$ for y. (Watch Video 22.)